BK BIRLA CENTRE FOR EDUCATION

SARALA BIRLA GROUP OF SCHOOLS
SENIOR SECONDARY CO-ED DAY CUM BOYS' RESIDENTIAL SCHOOL

PRE BOARD-1, (2024-25)

MATHEMATICS (041)

Class: XII Science — Duration: 3 Hour
Date: 16/11/24 — Max. Marks: 80
Admission Number: __________ — Roll number: ____

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment of 4 marks each with sub parts.

Section –A (Multiple Choice Questions)
Each question carries 1 mark

1 Let f:R→R be a function defined by $f(x)=x^3+1$, then f is

a) Injective but not surjective b) Surjective but not injective

c) Bijective d) neither injective nor surjective

2 The value of expression $2\sec^{-1}2+\sin^{-1}\frac{1}{2}$ is

a) $\frac{\pi}{6}$ b) $\frac{5\pi}{6}$ c) $\frac{7\pi}{6}$ d) 1

3 If $A=\begin{bmatrix}1 & a\\ 0 & 1\end{bmatrix}$, the A^n (where $n\in N$) equals

a) $\begin{bmatrix}1 & na\\ 0 & 1\end{bmatrix}$ b) $\begin{bmatrix}1 & n^2a\\ 0 & 1\end{bmatrix}$ c) $\begin{bmatrix}1 & na\\ 0 & 0\end{bmatrix}$ d) $\begin{bmatrix}n & na\\ 0 & n\end{bmatrix}$,

4 If $A=\begin{bmatrix}1 & 2 & x\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & -2 & y\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ and $AB = I_3$, then x+y equals

a) 0 b) -1 c) 2 d) none of these

5 If $A=\begin{bmatrix}1 & 3\\ 2 & 1\end{bmatrix}$, then $|A^2-2A|=$

a) 5 b) 25 c) -5 d) -25

6 If A and B are square matrices of order 3 such that AB = 6I. If $|A|=12$, the $|B|$ equals

a) $\frac{1}{2}$ b) 2 c) 18 d) 54

7 If the function $f(x)=\begin{cases}5x-4, if\ 0<x\le 1\\ 4x^2+3bx, if\ 1<x<2\end{cases}$ is continuous at every point of its domain, then the value of b is

a) -1 b) 0 c) 1 d) none of these

8 Given $f(x) = 4x^8$, then

a) $f'(\frac{1}{2}) = f'(-\frac{1}{2})$ b) $f(\frac{1}{2}) = -f'(-\frac{1}{2})$ c) $f(-\frac{1}{2}) = f(-\frac{1}{2})$ d) $f(\frac{1}{2}) = f'(-\frac{1}{2})$

9 Let $f(x) = x^3+3x^2-9x+2$. Then f(x) has

a) Maximum at x=1 b) Minimum at x=1

c) neither a maximum nor a minimum at x=-3 d) none of these

10 $\int \frac{(log x)^5}{x}$ dx is equal to

a) $\frac{log x^6}{6} + c$ b) $\frac{(log x)^6}{6} + c$ c) $\frac{(log x)^6}{3x^2} + c$ d) none of these

11 If $\int_0^{40} \frac{dx}{2x+1}$ = log k, then the value of k is

a) 3 b) $\frac{9}{2}$ c) 9 d) none of these

12 The area bounded by $y= 2-x^2$ and x+y=0 is

a) 7/2 sq.units b) 9/2 sq.units c) 9 sq.units d) none of these

13 The area bounded by the parabola $x=4-y^2$ and y-axis, in sq. units is

a) 3/32 b) 32/3 c) 33/2 d) 16/3

14 If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-zero vectors, no two of which are collinear and the vector $\vec{a} + \vec{b}$ is collinear with $\vec{c}$, $\vec{b} + \vec{c}$ is collinear with $\vec{a}$, the $\vec{a} + \vec{b} + \vec{c}$ equals

a) $\vec{a}$ b) $\vec{b}$ c) $\vec{c}$ d) none of these

15 The order and degree of the differential equation $\left(\frac{dy}{dx}\right)^5 + 3xy\left(\frac{d^3y}{dx^3}\right)^2 + y^2\left(\frac{d^2y}{dx^2}\right)^3 = 0.$

a) 2,3 b) 3,2 c) 3,5 d) 1,5

16 A dice is thrown twice, the probability of occurring of 5 at least once is

a) $\frac{11}{36}$ b) $\frac{7}{12}$ c) $\frac{35}{36}$ d) none of these

17 The lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x-1}{-2} = \frac{y-2}{-4} = \frac{z-4}{-6}$ are

a) Coincident b) Skew c) Intersecting d) Parallel

18 A and B are two events such that P (A) =0.25 and P (B) = 0.50. The probability of both happening together is 0.14. The probability of both A and B not happening is

a) 0.39 b) 0.25 c) 0.11 d) none of these

Assertion and Reasoning questions: In the following two questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(A) Both A and R are true and R is the correct explanation of A.
(B) Both A and R are true and R is not the correct explanation of A.
(C) A is true but R is false.
(D) *A is false but R is true.*

19 Assertion (A): If a line makes an angle α, β *and* γ with the coordinate axes, then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 1$
Reason (R): If l, m, n are direction cosines of a line, then $l^2+m^2+n^2 =1$.

20 Assertion (A): The general solution of the differential equation $\frac{dy}{dx}$= 1+2($\frac{y}{x}$) is x+y=Cx^2.
Reason (R): Correct substitution for the solution of the differential equation $\frac{dy}{dx}$ =f ($\frac{y}{x}$) is y=vx.

Section –B

[This section comprises of very short answer type questions (VSA) of 2 marks each]

21 Show that the function f: R→ R given by f(x) = x^3+x is bijection.

22 Evaluate the following cos (sin$^{-1}\frac{3}{5}$).

OR

Find the principal value of $\tan^{-1}\left\{2\sin(4cos^{-1}\frac{\sqrt{3}}{2})\right\}$.

23 The Find the interval in which f(x) = -x^2 -2x+15 is increasing or decreasing.

24 Evaluate: $\int \frac{2x}{\sqrt[3]{x^2+1}}dx$

OR

$Evaluate$: $\int_2^4 |x-3|dx$.

25 If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular unit vectors, find$\left|2\vec{a}+\vec{b}+\vec{c}\right|$.

Section – C

[This section comprises of short answer type questions (SA) of 3 marks each]

26 If x^y = y^x, Prove that$\frac{dy}{dx} = \frac{y(xlogy-y)}{x(ylogx-x)}$.

27 Find the integral of $\int \frac{x^3}{x^4+3x^2+2}dx$.

OR

$Evaluate$ $\int \left(x-\frac{1}{x}\right)^3dx$.

28 Solve the following differential equation: x^2(y+1) dx + y^2(x-1) dy =0.

29 Find the probability distribution of the number of tails in the simultaneous tosses of three coins.

OR

A man is known to speak truth 3 out of 4 times. He throws a die and report that it is a six.

30 Find the angle between unit vectors $\vec{a}$ and $\vec{b}$ so that $\sqrt{3}\ \vec{a}$ - $\vec{b}$ is also a unit vector.

31 Find the shortest distance between the lines $\vec{r}$= (4$\hat{i}$ -$\hat{j}$) +α($\hat{i}$ +2$\hat{j}$ +3$\hat{k}$) and $\vec{r}$= ($\hat{i}$ -$\hat{j}$ +2$\hat{k}$) +β(2$\hat{i}$ +4$\hat{j}$ -5$\hat{k}$).

Section –D

[This section comprises of long answer type questions (LA) of 5 marks each]

32 Let N be the set of all natural numbers and let R be a relation on NXN, defined by (a,b) R(c,d) such that ad=bc for all (a,b) , (c,d) ∈NXN. Show that R is an equivalence relation on NXN.

33 If A=$\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 1 \\ 1 & -3 & 1 \end{bmatrix}$, find $A^{-1}$and hence solve the system of linear equations: x+2y+z =4, -x+y+z = 0, x-3y+z= 2.

34 Find the area of the region included between the parabola $4y = 3x^2$ and the line 3x-2y+12=0.

OR

Find the area of the region $\{(x,y): y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$

35 Find graphically, the maximum value of Z= 2x+5y, subject to constraint 2x+4y≤ 8, $3x + y \leq 6, x + y \leq 4, x \geq 0, y \geq 0$.

OR

A mill owner buys two types of machines A and B for his mill. Machine A occupies 1000sq. m of area and requires 12 men to operate, while machine B occupies 1200sq. m area and requires 8 men to operate it. The owner has 7600 aq. M of area available and 72 men to operate machines. If machine A produces 50 units and machine B produces 40 units daily, how many machines of each type should he buy to maximise the daily output? Use linear programming to find the solution.

Section –E

[This section comprises of 3 case- study/passage based questions of 4 marks each with sub parts. The all three case study questions have three sub parts (i), (ii), (iii) of marks 1, 1, 2 respectively.

36 A firm produces two product P_1 and P_2 passing through two machines M_1 and M_2 before completion. Machine M_1 can produce either 8 units of P_1 or 10 units of P_2 per hour. Machine M_2 can produce 12 units of either product per hour. Per week machines M1 and M2 are available for 33 hours and 25 hours respectively. Based on the above information, answer the following:

(A) How many units of product P_1 are produced?
(B) How many units of product P_2 are produced?
(C) What is the total production per week of the firm?.

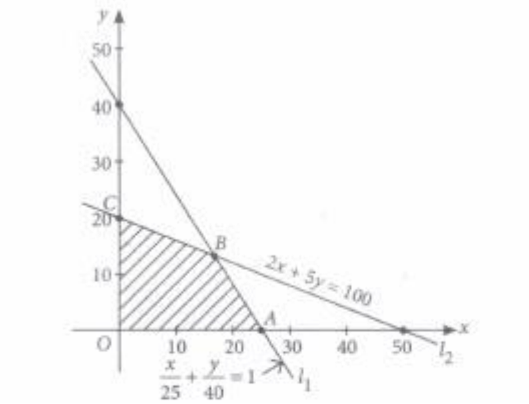
37 Engine displacement is the measure of the cylinder volume swept by all the pistons of piston engine. The piston moves inside the cylinder bore, the cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area $75\pi cm^2$. Based on the above information, answer the following questions:

A) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of radius r.

B) Find $\frac{dv}{dr}$.

C) Find the radius of cylinder when its volume is maximum.

38 Deepa rides her car at 25 km/hr, she has to spend Rs. 2 per km on diesel and if she rides it at a faster speed of 40 km/hr, the diesel cost increases to Rs. 5 per km. She has Rs. 100 to spend on diesel. Let she travels x kms with speed 25 km/hr and y kms with speed 40 km/hr. The feasible region for the LPP is shown below:
Based on the above information, answer the following questions



(i) What is the point of intersection of line l_1 and l_2?

(ii) What are the corner points of the feasible region shown in above graph?

(iii) If Z = x + y be the objective function and max Z = 30. What is the corner point where it gives maximum value?

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